Microscopic Simulation and Calibration of a Large-Scale Metropolitan Network: Issues and Proposed Solutions

Jianhe Du, Ph.D., P.E.
Virginia Tech Transportation Institute
3500 Transportation Research Plaza
Blacksburg, VA 24061
Phone: (540) 231-2673
Fax: (540) 231-1555
jdu@vtti.vt.edu

Hesham A. Rakha, Ph.D., P.Eng. (Corresponding author)
Charles E. Via, Jr. Department of Civil and Environmental Engineering
Virginia Tech Transportation Institute
Virginia Polytechnic Institute and State University
3500 Transportation Research Plaza
Blacksburg, VA 24061
Phone: (540) 231-1505
Fax: (540) 231-1555
hrakha@vt.edu

Ahmed Elbery
Virginia Tech Transportation Institute
3500 Transportation Research Plaza
Blacksburg, VA 24061
ahmed_elbery@mena.vt.edu

Matthew Klenk, Ph.D.
Palo Alto Research Center (PARC), a Xerox company
3333 Coyote Hill Road
Palo Alto, CA 94304 USA
Klenk@parc.com.

Word count: 5,100 (text) + 750 (3 figures) + 750 (3 tables) = 6,600 words

Submitted to the 97th Annual Meeting of Transportation Research Board

ABSTRACT

Microscopic traffic simulation has been used extensively to study network-wide congestion, traffic operations, traffic incidents, vehicle emissions, the performances of newly built transportation facilities and the effectiveness of traffic improvement projects. Because of the nature of microscopic simulation, it is typically used to study relatively small networks in which the level of demand is not too high and the road network is not large. In this study, INTEGRATION, a microscopic simulation tool, was used to model the Greater Los Angeles Area, a metropolitan area with a population of more than three million. To overcome the computational challenges associated with typical large-scale microscopic traffic simulation, the network was divided into five sub-networks with each network run on a different core, and the input demand file was also partitioned to account for connectivity between sub-networks. The results show that it is completely feasible to microscopically simulate large-scale networks. The findings are significant because they expand the applicability of microscopic simulation tools to large networks, which could only be modeled macroscopically or mesoscopically before. The agent-based microscopic results obtained can provide significantly more detailed vehicle-by-vehicle movement data that are expected to dramatically enhance the data of large-scale network simulations.

INTRODUCTION AND PROBLEM STATEMENT

Microscopic traffic simulation is a powerful tool that can track the movements of individual vehicles and recording detailed driver behavior, including car-following, lane-changing, and gap acceptance behavior, so that the traffic status of a network can be described based on the results generated from the simulation at a very detailed level. Microscopic traffic simulation has evolved significantly since its introduction in the 1990s, particularly due to the development of computer technology and programming tools. According to the FHWA, more than 30 microscopic simulation tools have been developed *[1])* and are widely used in all stages of transportation planning, design, management, analysis, and improvement as well as in applications in related fields such as evacuation and environmental impact analysis. Countless scientific studies have been conducted using microscopic traffic simulations.

Microscopic simulation is often used for relatively small-sized traffic modeling, where the network has a limited number of links, and the number of vehicles to be modeled is not too large. This is because microscopic traffic simulation software typically tracks the agent being modeled at a relatively high frequency (e.g., 1/10 second when using INTEGRATION) while concurrently calculating the agent's detailed location and behavior. When the size of the network or traffic demand increases, the associated computational workload increases exponentially. Applying microscopic simulation to a large network requires an extraordinary computational capability because of the large number of vehicles in the network, the large number of traffic signals that need to be optimized, and the large number of links and detector locations that should be processed. Additionally and most importantly, a larger network size corresponds to a larger routing tree and a longer time needed to build this tree. Because of the complexity of Dijkstra's shortest path algorithm: O (V^2), where V is the number of nodes in the network, the simulation time increases quadratically with the size of a network.

When a large-scale metropolitan area needs to be modeled in a simulation environment, mesoscopic, macroscopic, or hybrid traffic simulation models are usually selected to ease the computational burden. Burghout et al. used a hybrid mesoscopic-microscopic model that applies microscopic simulation to areas of specific interest, while simulating a large surrounding network in lesser detail with a mesoscopic model in two case studies *[2]*. Balakrishna et al. modeled the I-5 corridor, including 760 nodes and 972 links, using TransModeler where a certain of links of interest were modeled microscopically while the majority of the network was simulated macroscopically *[3]*. Zhao and Sadek modeled the buffalo area, network sized with 2,000 nodes and 3,000 links, using TRANSIMS during a lake-effect snow storm *[4]*. Meister et al. used MATSim-T to simulate a large area of Switzerland with more than 6 million synthetic persons and 1 million links *[5]*. Kotsialos et al. used METANET, a macroscopic simulator, to simulate a large-scale motorway network around Amsterdam*[6]*. The network has 654 links, totaling to 143 km. Sewall et al. used a hybrid simulator to interactively simulate a virtual large-scale network *[7]*. Zahng et al. modeled the demand of city of Shanghai, a city of about 20 million agents, 50,000 links, and 90,000 destinations, using MATSIM *[8]*. Zitzow et al. developed a hybrid simulation model to model the Twin Cities of Minnesota, where there are 19,350 links and 8,403 nodes *[9]*.

However, such modeling methods cannot capture the details of traffic status needed for research and practice over a large area. The best way to capture all the details is to conduct the simulation microscopically. Researchers have to balance between reduced simulation detail and the potential size of the simulation modeling network. Possible solutions to this dilemma are: (1) distributed parallel processing *[10-18]*, which entails distributing the processing over multiple single-processor machines to allow for the scaling of performance by demand for large-scale computations; and (2) simplifying the network by including only major roads and arterials. This second solution allows the simulation to cover a relatively large area but limits the details of results generated by the microscopic simulation *[19-22]*. Based on previous studies, to apply microscopic simulations in large-scale metropolitan areas, challenges related to high computational load and accurate modeling input data need to be addressed *[23]*.

This paper is based on a large-scale study that optimized the decisions of travelers, including travel mode, departure time, and route choice, to minimize vehicle fuel consumption and emissions in a large metropolitan area (Los Angeles, California). Details of travelers' choices are needed to accurately estimate fuel consumption and optimize system energy usage. However, the tremendous size of the simulation area prevents the direct microscopic simulation of the system. To solve this dilemma, a method was designed to partition the network into sub-networks. Simulation and calibration were then conducted individually within each sub-network. This method successfully bridged the gap between microscopic simulation and the large network size. The simulation input files and process were carefully calibrated. The simulation results were compared against observed traffic volume data. The results indicate that the new methodology can effectively solve the problems associated with simulating large-scale networks. This method is potentially general and can be used to model other large-scale metropolitan areas with high demand and complicated road network configurations.

MODELING METHODOLOGY

The modeling area covers the Greater Los Angeles Area including the downtown LA area and the immediate vicinity, totaling approximately 500 square miles. The original network with all levels of road links included more than 180,000 road links. INTEGRATION, an agent-based microscopic traffic assignment and simulation software, was used as the simulation tool in this study. To obtain a satisfactory result from microscopic simulation modeling, the following criteria need to be satisfied: a well-calibrated modeling tool with embedded car-following and route choice models; accurate input data including network configuration and an origin-destination (OD) matrix; and a powerful simulation environment to support the extraordinary model size. The following section presents the strategies used to address each of these criteria.

Modeling Tool: INTEGRATION

INTEGRATION was developed in the late 1980s and continues to be developed at VTTI *[24-26]*. INTEGRATION is an integrated simulation and traffic assignment model that creates individual vehicle trip departures based on an aggregated time-varying O-D matrix. In consideration of traffic control devices and gap acceptance, INTEGRATION moves vehicles along the network in accordance with embedded preset traffic assignment models and the Rakha-Pasumarthy-Adjerid (RPA) car-following model. RPA was developed at VTTI and has been calibrated and improved with different data sources, including data from the 100-car naturalistic driving study (12 billion observations) *[27, 28]*. The RPA model is consistent with the steady-state car-following model and is constrained by characteristics including vehicle power and traction, aerodynamic drag and rolling friction, and current momentum and grade resistance *[27, 29-33]*. Calibration results of the RPA model proved that the model is consistent with naturalist driving behaviors. Ten different routing options are available in INTEGRATION. Sub-population feedback assignment was selected in this study; this option divided the entire driver group into five sub-groups. The paths for 20% of the drivers were updated every 300 seconds, one sub-group at a time, based on the real-time measurement of link travel time. The simulation tracked the movement of individual vehicles every 0.1 s, allowing detailed analysis of lane-changing movements and shockwave propagation.

The simulation also permitted considerable flexibility in representing spatial and temporal variations in traffic conditions *[29, 34, 35]*. INTEGRATION parameters are calibrated to support the large-scale networks by extending the allowable memory. The routing tree size is adjusted to support the largest sub-network. INTEGRATION was chosen as a base model in this study because of these unique features.

Model Construction

One largest challenge in large-scale microscopic simulation is obtaining all the needed input data, including the details of the network configuration (e.g., number of lanes, free-flow speeds, lane striping, traffic signal timing plans, intersection controls) and, most importantly, an accurate demand file that reflects the traffic congestion level and hot spots in the network. In this study, a unique data source and powerful estimation tool were employed to address this challenge.

Network Configuration

Network coding was based on the associate attributes in the original GIS shapefile, where two variables, the speed class and function class, define the range of capacity and free-flow speed. Also included in the original file is a lane category variable that gives the number of lanes. The parameters of the network were set in accordance with the Highway Capacity Manual *[36]*). An automatic coding algorithm was developed in MatLab to convert the basic network input files from ArcGIS shapefiles to the ASCII format needed by INTEGRATION. Manual inspection and updating were conducted to ensure the accuracy of coding with the aid of Google Aerial Maps accounting for the upgrades and changes in the road network since the shapefile was created. Final network parameter settings are listed in TABLE 1. The traffic control types (traffic signals, stop signs, or yield signs) at each intersection were extracted from OpenStreetMap data *[37])*. Google Maps was used as a supplemental tool at locations where the traffic control data were missing. Traffic signal timing phase lengths and cycles are typically designed and managed by local traffic agencies, and it is infeasible to contact these agencies individually to obtain traffic signal timing plans over the entire area. Consequently, the traffic signal timing plans were optimized by INTEGRATION at the frequency of 300 seconds.

Function Class	**Capacity (Veh/Hour/Lane)**	**Jam Density (Veh/Lane/KM)**	**Speed Category**	**Free Flow Speed (KM/H)**	**Speed at Capacity (KM/H)**
1	2400	180	**2**	110	94
2	2400	180	**3**	90	76
3	2300	180	**4**	70	60
4	2100	180	**5**	45	38
5	2000	180	**6**	40	34
			7	30	25
			8	15	12

TABLE 1 Attributes of Road Links of All Levels

Static O-D Estimation

A static O-D demand file was generated using QueensOD *[38]*, a software application developed by VTTI researchers. QueensOD estimates the most-likely time-dependent static O-D using observed link traffic flows, observed link turning movement counts, link travel times, and a seed matrix. QueensOD iteratively minimizes the error between the observed link volumes and estimated link flow to generate a most-likely O-D matrix that is as close as possible to the seed matrix. FIGURE 1 illustrates the flow of QueensOD. According to Aerde et al. *[39]* and Rakha *[40]*, the objective function for estimating the static O-D using QueensOD is given as

$$Z\left(T_{ij}(t)\right) = \frac{T(t)!}{\prod_{ij} T_{ij}(t)!} \prod_{ij} \left(\frac{\tau_{ij}(t)}{\sum_{ij} \tau_{ij}(t)}\right)^{T_{ij}(t)}, \qquad (1)$$

where Z is the entropy that is maximized by the optimum O-D matrix $T_{ij}(t)$;
$\tau_{ij}(t)$ is the seed O-D matrix.

The input files needed for QueensOD include links, nodes, observed link flows, and a seed matrix. In this study, the median of the traffic count data for ten randomly selected Tuesdays and Wednesdays in 2014, which were provided by the Caltrans Performance Measurement System (PEMS) *[41])*, were used as the input observed link flow data for QueensOD. The seed file, which is used as the starting point for demand estimation, was generated from the planning model input data obtained from the Southern California Association of Governments (SCAG). SCAG planning data originally included five-time periods (24 hours) for weekdays with 4,109 internal zones and 83 external zones. The area covered by the SCAG planning model was spatially joined with the network of the simulation model. The traffic analysis zones (TAZs) were disaggregated or aggregated depending on the spatial distribution of the TAZ compared to the location of the simulation zone, as shown in FIGURE 2. The associated demand of TAZ was distributed over 1,100 zones.

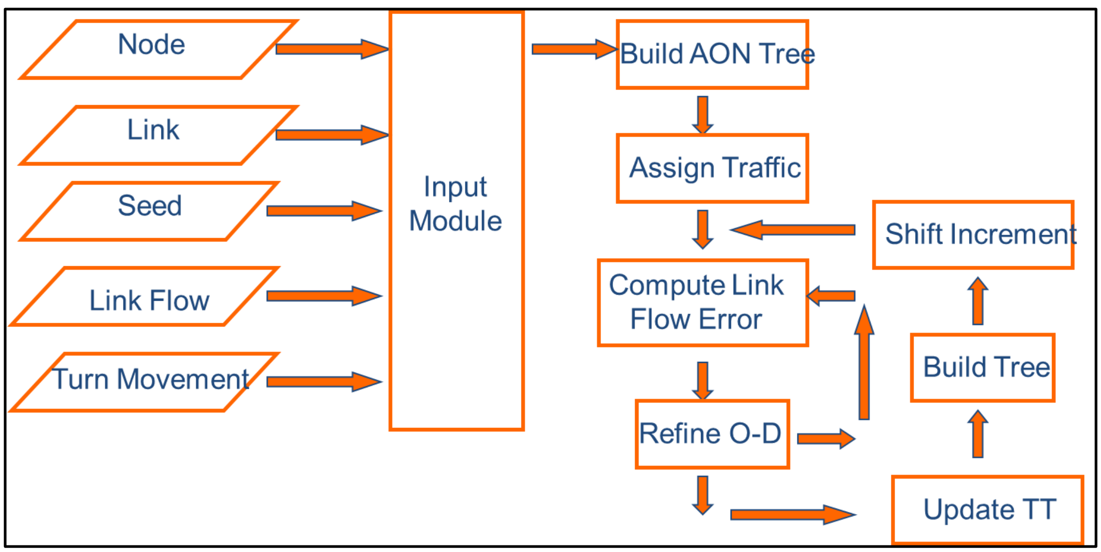

FIGURE 1 Process for Estimating Static O-D Matrix

FIGURE 2 TAZs and Zones

On average, the resulting global O-D had over 450,000 O-D pairs, totaling 400~500 thousand vehicles per hour. However, the static O-D tended to overestimate the demand, thus imposing an unrealistic burden on the simulation with extra counted trips. Consequently, the static O-D was adjusted to estimate the dynamic O-D.

Dynamic O-D Estimation

A precise estimation of the dynamic O-D matrix is a vital part of the simulation calibration. Since the estimated static O-D only provides an average demand for each O-D pair per time slice of simulation, it is not sufficient to capture network dynamics. When the network is over-congested, and the average travel time of each trip is greater than the time interval used to estimate the static O-D, there will be excess vehicles still traveling in the network when the previous modeling time slice ends. These trips need to be deducted from the static O-D matrices for the following modeling time slice. Dynamic O-D is a time-dependent O-D matrix that avoids over-estimating demand to account for the variation in traffic conditions over the analysis period.

Since INTEGRATION can trace the status of each vehicle every decisecond, it is possible to identify the trips retained by the end of the previous simulation time slice and remove those trips from the O-D matrix for the next time slice. A novel approach for estimating the dynamic O-D matrix from the static O-D matrix was used in this study. Using the static O-D matrix estimated by QueensOD as the starting point, several simulation runs were conducted for each analyzing time slice. Details of this dynamic O-D estimation can be found in Yang and Rakha *[42]*. Equation (2) was used for the estimation of dynamic O-D:

$$\begin{cases} T_{ij}^{t\,\prime} = T_{ij}^{t} - \alpha ER_{ij}^{t-1} & \forall\, T_{ij}^{t} \geq ER_{ij}^{t-1} \\ T_{ij}^{t\,\prime} = T_{ij}^{t} & \forall\, T_{ij}^{t} < ER_{ij}^{t-1} \end{cases}, \qquad (2)$$

where

$T_{ij}^{t\,'}$ is the updated trip number between origin i and destination j for time slice t;
T_{ij}^{t} is the original static demand between origin i and destination j for time slice t;
ER_{ij}^{t-1} is the en-route trips between origin i and destination j at the end of time slice $t-1$; and
α is the user-defined adjustment factor.
On average, the resulting dynamic O-D reduced the static O-D by 5% to 10%.

Network Division and Demand Splitting

INTEGRATION and the QueensOD software are built in FORTRAN to take advantage of its computational speed when dealing with large matrices. To maximize the simulation speed, both INTEGRATION and QueensOD are based on a set of shared modules with a set of shared static arrays. These shared static arrays define the size of the network and are statically allocated by the operating system. When working with a large network, these static arrays create an important limitation because the Windows operating system does not allow FORTRAN to allocate more than 2 GB in both the X86 and X64 architectures when using static arrays. Unfortunately, 2 GB is not sufficient for the large network in this study. To increase the memory allocation in Windows, it is necessary to use the 64-bit Windows version and to use dynamic arrays instead of static arrays. Currently, the majority of the machines are based on X64 architecture and run 64-bit Windows.

To overcome the above memory limitation and computational obstacle, the network was partitioned into five sub-networks, as shown in FIGURE 3. Accordingly, the network file (nodes, links, signals, and other input files) and the demand file need to be divided. The entire area was partitioned considering the similarity of the traffic conditions. For example, sub-network 3 includes the most congested downtown area. While it is relatively easy to divide the network files (the polygon file of the sub network is overlaid on top of the network links and nodes to identify links and nodes for each sub-network), the division of the demand file is more difficult since it involves the identification of routes to be used by each particular O-D pair. Fortunately, along with the global demand file generated using QueensOD, a tree file describes the up to five paths used by each O-D pair and the corresponding proportion of the O-D pair using the path. This tree file identifies the link-by-link route between each origin and destination. Up to five trees were generated during the static O-D estimation, and each tree file was assigned a weighting value. The tree file and the associated weight value were used to partition the network demand. The global demand was disaggregated into sub-network demands. Whenever a trip went in or out of a sub-network, it was broken into two individual trips, each occurring in a separate sub-network. The weight was used to distribute the trips between each O-D pair by routes recorded in the tree file. The origin and destination along with the time stamp of that trip was then written into a sub-network O-D matrix for that particular sub-network. The global demand O-D matrix was therefore disaggregated into five sub-groups.

The statistics for the resulting sub-network input files are given in TABLE 2. Each sub-network had 100–400 zones, 1,700–3,500 links, and 600–1,600 nodes. Traffic for three hours around the morning peak (7 am to 10 am) with one hour of preloading (6 am to 7 am) and three hours around the afternoon peak (4 pm to 7 pm) with one hour of preloading (3 pm to 4 pm) were simulated for the network. Approximately 2.1 and 2.3 million vehicles were simulated in the morning and afternoon peaks, respectively.

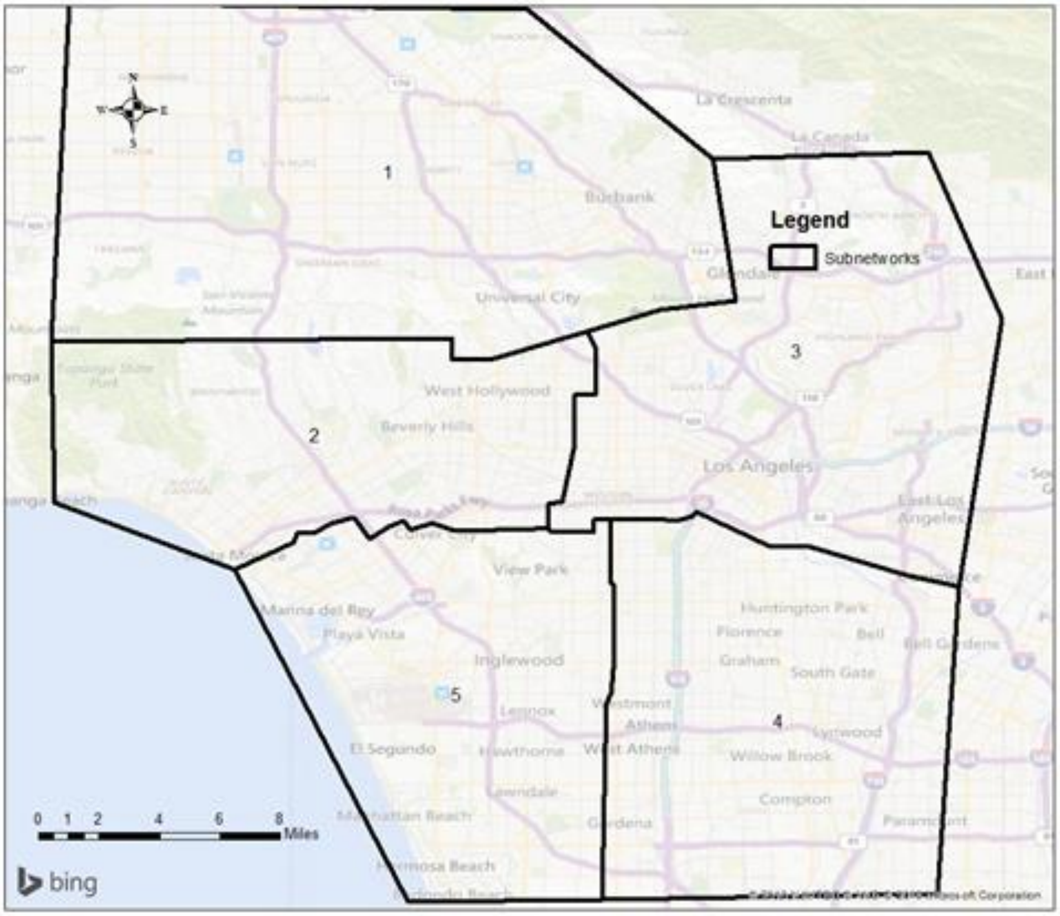

FIGURE 3 Network Partitioning

TABLE 2 O-D Pairs and Number of Trips

	Sub-network 1	Sub-network 2	Sub-network 3	Sub-network 4	Sub-network 5
Links	1,700	2,250	3,500	1,700	1,500
Nodes	740	1,000	1,600	740	650
Zones	145	199	405	234	195
AM O-D Pairs (Number of Trips)	44,000 (400,000)	220,000 (430,000)	270,000 (530,000)	97,000 (450,000)	105,000 (370,000)
PM O-D Pairs (Number of Trips)	40,000 (430,000)	192,000 (440,000)	260,000 (590,000)	95,000 (480,000)	91,000 (390,000)

CALIBRATION AND SIMULATION RESULTS

The calibration and simulation process involved iterations of running simulations, comparing the results with observed data, and modifying the parameters of the input files and embedding models to ensure the accuracy of the simulation results. Since INTEGRATION has been carefully calibrated for its embedded car-following, gap acceptance, and route choice models in multiple previous studies conducted by the authors, the calibration in this study only focused on adjustment of input parameters.

Modified parameters included speed, number of lanes, signal timing plans, and roadway lane striping. Because the simulation network did not include some of the lowest hierarchical roads that indeed distribute major traffic flow due to their special locations in certain highly populated areas (e.g., the University of California, Los Angeles or the Los Angeles International Airport), the network configuration in the simulation environment needed to be adjusted to accommodate for concentrated traffic flow using alternative minor roads. Accordingly, the traffic signal timings and other network parameters were adjusted during the calibration process.

The simulation results for each simulated hour were compared against the traffic count data. Since the observed traffic volumes themselves are not fixed values, the following procedures were adopted to evaluate the simulation results:

1. Twenty days of typical workday data (Tuesdays and Wednesdays) were randomly extracted from PEMS;
2. The median traffic volume was calculated for every hour in the morning peak (7 am to 10 am) and afternoon peak (4 pm to 7 pm);
3. The R-value for each day of observed data with respect to the median value was calculated;
4. The lowest R-value, which reflects the largest possible fluctuation in observed traffic volume, was identified as the baseline for comparison;
5. The simulation runs were conducted, and simulated traffic volumes were extracted from links that are listed as the locations where the loop detectors are placed.
6. The R-value for the simulated traffic volume and the median observed traffic count data was calculated.

These two sets of R-values are listed in TABLE 3. As can be seen from the table, all the sub-networks had comparable R-values for the simulation results and the observed volumes. This indicates that the temporal variation in observed traffic volume is in the same range as the difference between the simulated traffic volume and the median traffic volume. The results indicate that the simulation accurately modeled the traffic conditions in the modeling area.

TABLE 3 Calibrated Simulation Results

	Sub-network 1		**Sub-network 2**		**Sub-network 3**		**Sub-network 4**		**Sub-network 5**	
Time	***Sim-R***	***Obs-R***	***Sim-R***	***Obs-R***	***Sim-R***	***Obs-R***	***Sim-R***	***Obs-R***	***Sim-R***	***Obs-R***
7 – 8 AM	0.88	0.91	0.93	0.92	0.93	0.95	0.90	0.90	0.95	0.96
8 – 9 AM	0.85	0.90	0.94	0.81	0.93	0.95	0.95	0.86	0.94	0.95
9 – 10 AM	0.86	0.92	0.91	0.88	0.94	0.91	0.90	0.95	0.95	0.95
4 – 5 PM	0.88	0.93	0.96	0.88	0.91	0.95	0.86	0.96	0.92	0.96
5 – 6 PM	0.88	0.93	0.96	0.93	0.89	0.95	0.82	0.97	0.95	0.97
6 – 7 PM	0.71	0.90	0.95	0.95	0.90	0.95	0.84	0.96	0.94	0.97

DISCUSSION AND CONCLUSIONS

Microscopic simulation is a powerful tool for traffic studies. It is widely used in both academic research and practice by transportation administration agencies. The main limitation of microscopic simulation is associated with its primary advantage: being able to record the details of simulated vehicles including car-following, shockwave propagation, lane-changing behavior, and other data needed for in-depth traffic analysis. This level of detail creates a huge challenge related to the computational capabilities of computers. A common solution to this problem is distributed parallel processing, which can be expensive and complicated. While simplifying the network by selecting only skeleton arterials can also help, it sacrifices valuable information needed for research and traffic administration.

This study solves the problem by dividing a large-scale network into sub-networks and simulating each sub-network individually. In addition, this paper discusses the methods used to improve the accuracy of input data by integrating multiple network input sources, calibrating vital input network parameters, and estimating reliable dynamic O-D matrices from a static O-D matrix. The methodology and simulation results reported in this study are significant for the following reasons:

1) The methodology used to construct a large-scale network and calibrate network parameters is general in nature and thus are transferable.
2) The data sources used to estimate the O-D demand are easy to access. Planning data and observed traffic count data on freeways and major arterials are generally accessible. By combing these two data sources, QueensOD and INTEGRATION can be used jointly to accurately estimate dynamic O-D matrices.
3) Divide a large network into small sub-networks eases the calibration process and the computational burden of microscopic simulation. By simulating sub-networks in parallel, applying microscopic simulation to large networks is no longer infeasible.
4) The size of the network and demand modeled in this study is unprecedented. Previous studies using microscopic simulation only modeled either a smaller area or a simpler network with less vehicles.
5) The simulation results are highly accurate. The deviation of simulation results from the median values of observed traffic volume was comparable to the variation in observed traffic volumes themselves.

There are issues that were not discussed in this paper and are worthy of additional investigation. For example, the synchronization of the trips that travel across sub-networks is a separate research topic that will be done via a traffic simulation controller that will monitor and track vehicles across all networks. The traffic simulation controller is currently under development.

ACKNOWLEDGEMENTS

The information, data, or work presented herein was funded in part by the Advanced Research Projects Agency-Energy (ARPA-E), U.S. Department of Energy, under Award Number DE-AR0000612

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